

Chapman-Enskog expansion and relativistic dissipative hydrodynamics

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Starting from Boltzmann equation with relaxation time approximation for the collision term and using Chapman-Enskog expansion for distribution function close to equilibrium, we derive hydrodynamic evolution equations for the dissipative quantities directly from their definition. Although the form of the equations is identical to those obtained in traditional Israel-Stewart approaches employing Grad's 14-moment approximation and second moment of Boltzmann equation, the coefficients obtained are different. In the case of one-dimensional scaling expansion, we demonstrate that our results are in better agreement with numerical solution of Boltzmann equation.

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Relativistic fluid dynamics has been applied quite successfully to study and understand a wide range of collective phenomena observed in cosmology, astrophysics and the physics of high-energy, heavy-ion collisions. The earliest theoretical formulation of relativistic dissipative hydrodynamics also known as first-order theories (order of gradients), are due to Eckart [1] and Landau-Lifshitz [2]. However these theories, collectively called relativistic Navier-Stokes (NS) theories, involve parabolic differential equations and suffers from acausality and instability. The Chapman-Enskog (CE) expansion has been the most common method to obtain first-order hydrodynamics from Boltzmann Equation (BE) [3]. The derivation of second-order fluid-dynamics by Israel and Stewart (IS) from kinetic theory uses extended Grad's method [4]. The approach by Israel and Stewart may not guarantee stability but solves the acausality problem [5] at the cost of introducing two additional approximations: 14-moment approximation for the distribution function and use of second moment of BE to obtain evolution equations for dissipative quantities.

Grad's method, originally proposed for non-relativistic systems, was modified by Israel and Stewart so that it could be applicable in the relativistic case. In this extension, known as 14-moment approximation, the distribution function is Taylor expanded in powers of four-momenta around its local equilibrium value. Truncating the Taylor expansion at second-order in momenta results in 14 unknowns that has to be determined to describe the distribution function. This expansion implicitly assumes a converging series in powers of momenta. In addition, it is assumed that the order of expansion in 14-moment approximation (expanded as a series in momenta) coincides with that of gradient expansion of hydrodynamics. This is evident because Grad's approximation truncated at second-order in momenta is not consistent with second-order hydrodynamics.

Another assumption inherent in IS derivation is the choice of second moment of the BE to extract the equation of motion for the dissipative quantities. This choice is arbitrary in the sense that once the distribution func-

tion is specified, any moment of the BE will lead to a closed set of equations for the dissipative currents but with different transport coefficients. In fact, it has been pointed out in Ref. [6] that instead of this ambiguous choice of the second-moment of BE by IS, the dissipative quantities can be obtained directly from their definition.

In this communication, we present an alternate derivation of hydrodynamic equations for dissipative quantities which does not make use of both these assumptions. We revisit the CE expansion of the distribution function using BE in relaxation time approximation. Using this expansion, we derive the first and second-order equations of motion for the dissipative quantities from their definition. For the case of one-dimensional scaling expansion, we compare our results with the results obtained by using both 14-moment approximation as well as second moment of BE (IS equations). We demonstrate that our results are in better agreement with direct numerical solution of BE as compared to those obtained by using IS equations.

Fluid dynamics is best described as a long-wavelength, low-frequency limit of an underlying microscopic theory. BE governs the evolution of single particle phase-space distribution function $f \equiv f(x, p)$ which provides a reliably accurate description of the microscopic dynamics in the dilute limit. With this motivation, our starting point for the derivation of hydrodynamic equations is relativistic BE with relaxation time approximation for the collision term [7]

$$p^\mu \partial_\mu f = -\frac{u \cdot p}{\tau_R} (f - f_0), \quad (1)$$

where, p^μ is the particle four-momentum, u_μ is the fluid four-velocity and τ_R is the relaxation time. We define the scalar product $u \cdot p \equiv u_\mu p^\mu$. With $f \rightarrow \bar{f}$ and $f_0 \rightarrow \bar{f}_0$, Eq. (1) describes the evolution of distribution function for antiparticles \bar{f} . The equilibrium distribution functions for Fermi, Bose, and Boltzmann particles ($r = 1, -1, 0$), is $f_0 = [\exp(\beta u \cdot p - \alpha) + r]^{-1}$ and for antiparticles is $\bar{f}_0 = [\exp(\beta u \cdot p + \alpha) + r]^{-1}$. Here, $\beta = 1/T$ is the inverse temperature and $\alpha = \mu/T$ is the ratio of chemical potential to temperature.

In the CE expansion, the particle distribution function is expanded about its equilibrium value in powers of space-time gradients.

$$f = f_0 + \delta f, \quad \delta f = \delta f^{(1)} + \delta f^{(2)} + \dots, \quad (2)$$

where $\delta f^{(1)}$ is first-order in gradients, $\delta f^{(2)}$ is second-order, etc. The Boltzmann equation, (1), in the form $f = f_0 - (\tau_R/u \cdot p) p^\mu \partial_\mu f$, can be solved iteratively as [8]

$$f_1 = f_0 - \frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_0, \quad f_2 = f_0 - \frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_1, \quad \dots \quad (3)$$

where $f_1 = f_0 + \delta f^{(1)}$ and $f_2 = f_0 + \delta f^{(1)} + \delta f^{(2)}$. To first and second-order in gradients, we obtain

$$\delta f^{(1)} = -\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu f_0, \quad (4)$$

$$\delta f^{(2)} = \frac{\tau_R}{u \cdot p} p^\mu p^\nu \partial_\mu \left(\frac{\tau_R}{u \cdot p} \partial_\nu f_0 \right). \quad (5)$$

The above treatment to obtain δf is valid for $\delta \bar{f}$ (antiparticles) as well.

For the sake of comparison, we also write down the Grad's 14-moment expansion [9] in orders of momenta as suggested by IS [4] in orthogonal basis [10],

$$\delta f = f_0 \tilde{f}_0 (\lambda_\Pi \Pi + \lambda_n n_\alpha p^\alpha + \lambda_\pi \pi_{\alpha\beta} p^\alpha p^\beta) + \mathcal{O}(p^3), \quad (6)$$

where, $\tilde{f}_0 = 1 - r f_0$ and λ_Π , λ_n , λ_π are determined from the definition of the dissipative quantities, Eqs. (9)-(11). Since hydrodynamics involves expansion in orders of gradients, hence for consistency, CE should be preferred over 14-moment approximation in derivation of hydrodynamic equations.

The conserved energy-momentum tensor and particle current can be expressed in terms of distribution function as [11]

$$T^{\mu\nu} = \int dp p^\mu p^\nu (f + \bar{f}) = \epsilon u^\mu u^\nu - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu},$$

$$N^\mu = \int dp p^\mu (f - \bar{f}) = n u^\mu + n^\mu, \quad (7)$$

where $dp = g d\mathbf{p}/[(2\pi)^3 \sqrt{\mathbf{p}^2 + m^2}]$, g and m being the degeneracy factor and particle rest mass. In the tensor decompositions, ϵ, P, n are respectively energy density, pressure, net number density, and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projection operator on the three-space orthogonal to the hydrodynamic four-velocity u^μ defined in the Landau frame: $T^{\mu\nu} u_\nu = \epsilon u^\mu$. The metric tensor is $g^{\mu\nu} \equiv \text{diag}(+, -, -, -)$. The bulk viscous pressure (Π), shear stress tensor ($\pi^{\mu\nu}$) and particle diffusion current (n^μ) are the dissipative quantities.

Energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$ and current conservation, $\partial_\mu N^\mu = 0$, yields the fundamental evolution equations for n, ϵ and u^μ

$$\begin{aligned} \dot{\epsilon} + (\epsilon + P + \Pi)\theta - \pi^{\mu\nu} \nabla_{(\mu} u_{\nu)} &= 0, \\ (\epsilon + P + \Pi)\dot{u}^\alpha - \nabla^\alpha (P + \Pi) + \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu} &= 0, \\ \dot{n} + n\theta + \partial_\mu n^\mu &= 0. \end{aligned} \quad (8)$$

We use the standard notation $\dot{A} = u^\mu \partial_\mu A$ for co-moving derivative, $\nabla^\alpha = \Delta^{\mu\alpha} \partial_\mu$ for space-like derivative, $\theta = \partial_\mu u^\mu$ for expansion scalar and $A^{(\alpha} B^{\beta)} = (A^\alpha B^\beta + A^\beta B^\alpha)/2$ for symmetrization.

Even if the equation of state relating ϵ and P is provided, the system of Eqs. (8) is not closed unless the dissipative quantities Π, n^μ and $\pi^{\mu\nu}$ are specified. To obtain the expressions for these dissipative quantities, we write them using Eq. (7) in terms of away from equilibrium part of the distribution functions ($\delta f, \delta \bar{f}$) as

$$\Pi = -\frac{\Delta_{\alpha\beta}}{3} \int dp p^\alpha p^\beta (\delta f + \delta \bar{f}), \quad (9)$$

$$n^\mu = \Delta^\mu_\alpha \int dp p^\alpha (\delta f - \delta \bar{f}), \quad (10)$$

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int dp p^\alpha p^\beta (\delta f + \delta \bar{f}), \quad (11)$$

where $\Delta^{\mu\nu}_{\alpha\beta} = [\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha - (2/3) \Delta^{\mu\nu} \Delta_{\alpha\beta}]/2$.

The first-order dissipative equations can be obtained from Eqs. (9)-(11) using $\delta f = \delta f^{(1)}$ from Eq. (4)

$$\Pi = -\frac{\Delta_{\alpha\beta}}{3} \int dp p^\alpha p^\beta \left[-\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu (f_0 + \bar{f}_0) \right], \quad (12)$$

$$n^\mu = \Delta^\mu_\alpha \int dp p^\alpha \left[-\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu (f_0 - \bar{f}_0) \right], \quad (13)$$

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int dp p^\alpha p^\beta \left[-\frac{\tau_R}{u \cdot p} p^\mu \partial_\mu (f_0 + \bar{f}_0) \right]. \quad (14)$$

Assuming the relaxation time τ_R to be independent of four-momenta, the integrals in Eqs. (12)-(14) reduce to

$$\Pi = -\tau_R \beta_\Pi \theta, \quad n^\mu = \tau_R \beta_n \nabla^\mu \alpha, \quad \pi^{\mu\nu} = 2\tau_R \beta_\pi \sigma^{\mu\nu}, \quad (15)$$

where $\sigma^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \nabla^\alpha u^\beta$. The coefficients β_Π , β_n and β_π are found to be

$$\begin{aligned} \beta_\Pi &= \frac{1}{3} (1 - 3c_s^2) (\epsilon + P) - \frac{2}{9} (\epsilon - 3P) \\ &\quad - \frac{m^4}{9} \langle (u \cdot p)^{-2} \rangle_{0+}, \end{aligned} \quad (16)$$

$$\beta_n = -\frac{n^2}{\beta(\epsilon + P)} + \frac{2 \langle 1 \rangle_{0-}}{3\beta} + \frac{m^2}{3\beta} \langle (u \cdot p)^{-2} \rangle_{0-}, \quad (17)$$

$$\beta_\pi = \frac{4P}{5} + \frac{\epsilon - 3P}{15} - \frac{m^4}{15} \langle (u \cdot p)^{-2} \rangle_{0+}, \quad (18)$$

where $\langle \dots \rangle_{0\pm} = \int dp (\dots) (f_0 \pm \bar{f}_0)$, and $c_s^2 = (dP/d\epsilon)_{s/n}$ is the velocity of sound squared (s being the entropy density). It is interesting to note that these coefficients are in perfect agreement with those obtained in the Ref. [6] in which the evolution equations are derived directly from their definition. This is due to the fact that in Ref. [6], the coefficients β_Π , β_n and β_π , are associated with first-order terms and do not involve 14-moment approximation. In the massless limit, $\beta_\pi \equiv \eta/\tau_\pi = 4P/5$ is also in agreement with that obtained in Ref. [8] employing CE expansion in BE with medium-dependent masses.

To obtain second-order equations, we find that CE expansion for the distribution function does not support derivation of hydrodynamic evolution equations from arbitrary moment choice of BE. Using the definition of dissipative quantities to obtain their evolution equations comes naturally when employing CE expansion as demonstrated while deriving first-order equations (15). Second-order evolution equations can be obtained directly by substituting $\delta f = \delta f^{(1)} + \delta f^{(2)}$ from (4) and (5) in Eqs. (9)-(11). However, we follow the methodology discussed in Ref. [6] to derive second-order equations for the sake of convenience and clarity which will become evident in the following. We have confirmed that both these methods lead to the same result.

We express the evolution of the dissipative quantities using Eqs. (9)-(11) as

$$\dot{\Pi} = -\frac{\Delta_{\alpha\beta}}{3} \int dp p^\alpha p^\beta (\delta \dot{f} + \delta \dot{\bar{f}}), \quad (19)$$

$$\dot{n}^{(\mu)} = \Delta_\alpha^\mu \int dp p^\alpha (\delta \dot{f} - \delta \dot{\bar{f}}), \quad (20)$$

$$\dot{\pi}^{(\mu\nu)} = \Delta_{\alpha\beta}^{\mu\nu} \int dp p^\alpha p^\beta (\delta \dot{f} + \delta \dot{\bar{f}}), \quad (21)$$

where the notations $A^{(\mu)} = \Delta_\nu^\mu A^\nu$ and $B^{(\mu\nu)} = \Delta_{\alpha\beta}^{\mu\nu} B^{\alpha\beta}$ represents space-like and traceless symmetric projections respectively orthogonal to u^μ .

The co-moving derivative of the away from equilibrium part of the distribution function ($\delta \dot{f}$) can be obtained by rewriting the BE (1) in the form

$$\delta \dot{f} = -\dot{f}_0 - \frac{1}{u \cdot p} p^\gamma \nabla_\gamma f - \frac{\delta f}{\tau_R}, \quad (22)$$

and similarly for antiparticles. Using this expression for $\delta \dot{f}$ in the second-order evolution equations (19)-(21), we get

$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_R} &= \frac{\Delta_{\alpha\beta}}{3} \int dp p^\alpha p^\beta \left[\left(\dot{f}_0 + \frac{1}{u \cdot p} p^\gamma \nabla_\gamma f \right) \right. \\ &\quad \left. + (f_0 \rightarrow \bar{f}_0, f \rightarrow \bar{f}) \right], \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{n}^{(\mu)} + \frac{n^{(\mu)}}{\tau_R} &= -\Delta_\alpha^\mu \int dp p^\alpha \left[\left(\dot{f}_0 + \frac{1}{u \cdot p} p^\gamma \nabla_\gamma f \right) \right. \\ &\quad \left. - (f_0 \rightarrow \bar{f}_0, f \rightarrow \bar{f}) \right], \end{aligned} \quad (24)$$

$$\begin{aligned} \dot{\pi}^{(\mu\nu)} + \frac{\pi^{(\mu\nu)}}{\tau_R} &= -\Delta_{\alpha\beta}^{\mu\nu} \int dp p^\alpha p^\beta \left[\left(\dot{f}_0 + \frac{1}{u \cdot p} p^\gamma \nabla_\gamma f \right) \right. \\ &\quad \left. + (f_0 \rightarrow \bar{f}_0, f \rightarrow \bar{f}) \right]. \end{aligned} \quad (25)$$

It is clear that in Eqs. (23)-(25), the Boltzmann relaxation time τ_R can be replaced by the relaxation times for the individual dissipative quantities τ_Π , τ_n , τ_π . At this stage, it seems as though the three relaxation times τ_Π , τ_n , τ_π are all equal to τ_R . This is because the collision term in the BE (1) is written in relaxation time

approximation which does not entirely captures the microscopic interactions. This apparent equality vanishes if the first-order equation (15) is compared with the relativistic Navier-Stokes equations for dissipative quantities ($\Pi = -\zeta\theta$, $n^\mu = \lambda T \nabla^\mu \alpha$ and $\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$). The dissipative relaxation times are then obtained in terms of first-order transport coefficients ζ , λ , η which can be calculated independently taking in account the full microscopic behavior of the system [12, 13].

For the dissipative equations to be second-order in gradients the distribution function in right hand side of Eqs. (23)-(25) need to be computed only till first-order ($\delta f = \delta f^{(1)}$) via CE expansion. We have repeatedly used the first-order equations for the dissipative quantities, Eq. (15), to replace $\theta \rightarrow \Pi$, $\nabla^\mu \alpha \rightarrow n^\mu$ and $\sigma^{\mu\nu} \rightarrow \pi^{\mu\nu}$ such that the relaxation times appearing on the right hand side of Eqs. (23)-(25) are absorbed.

The second-order evolution equations of the dissipative quantities are finally obtained as

$$\begin{aligned} \dot{\Pi} &= -\frac{\Pi}{\tau_\Pi} - \beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \\ &\quad - \tau_{\Pi n} n \cdot \dot{u} - \lambda_{\Pi n} n \cdot \nabla \alpha - \ell_{\Pi n} \partial \cdot n, \end{aligned} \quad (26)$$

$$\begin{aligned} \dot{n}^{(\mu)} &= -\frac{n^{(\mu)}}{\tau_n} + \beta_n \nabla^\mu \alpha - n_\nu \omega^{\nu\mu} - \lambda_{nn} n^\nu \sigma_\nu^\mu - \delta_{nn} n^\mu \theta \\ &\quad + \lambda_{n\Pi} \Pi \nabla^\mu \alpha - \lambda_{n\pi} \pi^{\mu\nu} \nabla_\nu \alpha - \tau_{n\pi} \pi_\nu^\mu \dot{u}^\nu \\ &\quad + \tau_{n\Pi} \Pi \dot{u}^\mu + \ell_{n\pi} \Delta^{\mu\nu} \partial_\gamma \pi_\nu^\gamma - \ell_{n\Pi} \nabla^\mu \Pi. \end{aligned} \quad (27)$$

$$\begin{aligned} \dot{\pi}^{(\mu\nu)} &= -\frac{\pi^{(\mu\nu)}}{\tau_\pi} + 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\gamma^{(\mu} \omega^{\nu)\gamma} - \tau_{\pi\pi} \pi_\gamma^{(\mu} \sigma^{\nu)\gamma} \\ &\quad - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} - \tau_{\pi n} n^{(\mu} \dot{u}^{\nu)} \\ &\quad + \lambda_{\pi n} n^{(\mu} \nabla^{\nu)} \alpha + \ell_{\pi n} \nabla^{(\mu} n^{\nu)}, \end{aligned} \quad (28)$$

We remark that although the form of the evolution equations for dissipative quantities obtained here, Eqs. (26)-(28), are the same as those obtained in the previous calculations using both 14-moment approximation and second moment of BE [14], the coefficients obtained are different. In the following, we refer to the results in Ref. [14] as the IS results although the power counting scheme differs from the one employed originally by IS.

For the special case of a system consisting of single species of massless Boltzmann gas, we find that

$$\beta_\pi = \frac{4P}{5}, \quad \tau_{\pi\pi} = \frac{10}{7}, \quad \delta_{\pi\pi} = \frac{4}{3}; \quad (29)$$

whereas these coefficients as obtained via IS approach are [14]

$$\beta_\pi^{\text{IS}} = \frac{2P}{3}, \quad \tau_{\pi\pi}^{\text{IS}} = 2, \quad \delta_{\pi\pi}^{\text{IS}} = \frac{4}{3}. \quad (30)$$

In this limit, although the coefficient of $\pi^{\mu\nu} \theta$ are same for both the cases ($\delta_{\pi\pi} = \delta_{\pi\pi}^{\text{IS}}$), the coefficient of $\sigma^{\mu\nu}$ and $\pi_\gamma^{(\mu} \sigma^{\nu)\gamma}$ are different ($\beta_\pi \neq \beta_\pi^{\text{IS}}$, $\tau_{\pi\pi} \neq \tau_{\pi\pi}^{\text{IS}}$).

We note that CE expansion, as opposed to 14-moment approximation, can be done iteratively to arbitrarily

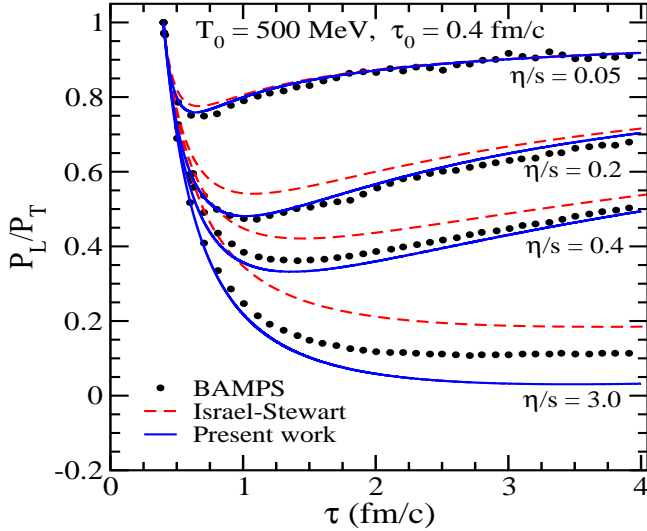


FIG. 1: (Color online) Time evolution of P_L/P_T in BAMPs (dots), IS (dashed lines), and the present work (solid lines), for isotropic initial pressure configuration ($\pi_0 = 0$).

higher orders. Hence using CE expansion, dissipative hydrodynamic equations of any order can in principle be derived. To obtain n th-order evolution equations for dissipative quantities, $\delta f = \delta f^{(1)} + \delta f^{(2)} + \dots + \delta f^{(n-1)}$ should be used in Eqs. (23)-(25). For instance, substitution of $\delta f = \delta f^{(1)} + \delta f^{(2)}$ from Eqs. (4) and (5) in Eqs. (23)-(25) will eventually lead to third-order evolution equations. Derivation of third-order hydrodynamics as outlined above is left for future work.

To demonstrate the numerical significance of the new coefficients derived here, we consider evolution in the boost invariant Bjorken case of a massless Boltzmann gas ($\epsilon = 3P$) at vanishing net baryon number density [15]. In terms of the co-ordinates (τ, x, y, η) , where $\tau = \sqrt{t^2 - z^2}$ and $\eta = \tanh^{-1}(z/t)$, the initial four-velocity becomes $u^\mu = (1, 0, 0, 0)$. For this scenario, $\Pi = n^\mu = 0$, and the evolution equations for ϵ , $\pi \equiv -\tau^2 \pi^{\eta\eta}$ reduces to

$$\frac{d\epsilon}{d\tau} = -\frac{1}{\tau}(\epsilon + P - \pi), \quad (31)$$

$$\frac{d\pi}{d\tau} = -\frac{\pi}{\tau} + \beta_\pi \frac{4}{3\tau} - \lambda \frac{\pi}{\tau}. \quad (32)$$

The second-order transport coefficients simplifies to

$$\lambda \equiv \frac{1}{3}\tau_{\pi\pi} + \delta_{\pi\pi} = \frac{38}{21}, \quad \lambda^{\text{IS}} = 2. \quad (33)$$

Initial temperature $T_0 = 500$ MeV at proper time $\tau_0 = 0.4$ fm/c are chosen to solve the coupled differential Eqs. (31) and (32). These values corresponds to LHC initial conditions [16]. We assume isotropic initial pressure configuration i.e. $\pi_0 = 0$. Fig. 1, shows the proper time dependence of pressure anisotropy which is defined as

$$\frac{P_L(\tau)}{P_T(\tau)} = \frac{P(\tau) - \pi(\tau)}{P(\tau) + \pi(\tau)/2}. \quad (34)$$

The solid and dashed lines represents our results and the results from IS theory, respectively. The dots corresponds to the results of a transport model, the parton cascade BAMPs [17, 18]. The calculations in BAMPs are performed with variable values for the cross section such that the shear viscosity to entropy density ratio is a constant.

We note that the results from IS theory always overestimates the pressure anisotropy as compared to the BAMPs results even for viscosities as small as $\eta/s = 0.05$. The figure also shows that our results are in better agreement to the transport results. For very high viscosity, $\eta/s = 3.0$, although at early times we have a better agreement with BAMPs as compared to IS, at later times there is a large deviation. This disagreement may be due to the relaxation time approximation for the collision term considered in Eq. (1).

Relaxation time approximation for the collision term assumes that the effect of the collisions is to restore the distribution function to its local equilibrium value exponentially. This is a very good approximation as long as the deviations from local equilibrium is small. For large η/s values, shear viscous pressure is also large. This in turn implies that the deviations from equilibrium is not small resulting in breakdown of the relaxation time approximation. It is important to note that large values of η/s (> 0.4) are not relevant to the physics of strongly coupled systems like Quark Gluon Plasma (QGP). The QGP formed at RHIC and LHC behaves as a near perfect fluid with a small estimated $\eta/s \approx 0.08 - 0.2$ [19, 20]. The fact that we get reasonably good agreement with BAMPs results for $\eta/s \leq 0.4$ (Fig. 1) also suggests that BE with relaxation time approximation for the collision term can be applied in understanding the hydrodynamic behaviour of QGP formed in relativistic heavy-ion collisions.

To summarize, we have presented a new derivation of relativistic second-order hydrodynamics from BE. We use Chapman-Enskog expansion for out of equilibrium distribution function instead of 14-moment approximation and derive evolution equations for dissipative quantities directly from their definitions rather than employing second moment of Boltzmann equation. In this new approach, we get rid of two powerful assumptions of IS kind of derivation which is 14-moment approximation and choice of second moment of BE. Although the form of the evolution equations remain the same, the coefficients are found to be different. Our results show a better agreement with the parton cascade BAMPs for the P_L/P_T evolution.

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